Name:________________________________

MA 4633 Section 01 | *Practice Exam 1* | *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution you will receive little or no credit!

1. Prove, using the N-ε equivalent definition of limit, that

$$\lim_{n\to\infty} \frac{3n+2}{6n+1} = \frac{1}{2} \ .$$

2. Prove, using the N-ε equivalent definition of limit, that

$$\lim_{n\to\infty} \frac{n+2}{7n+5} = \frac{1}{7} \ .$$

3. Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms. Let $c > 0$ be a real number such that

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c \ .$$

Prove that both series converge or both series diverge.

4. Suppose that $\sum_{n=1}^{\infty} a_n$ converges absolutely and $\{b_n\}$ is a bounded sequence of real numbers. Prove that

$$\sum_{n=1}^{\infty} a_n b_n$$

converges absolutely.

5. Let A and B be two compact subsets of $\mathbb{R}$. Prove that $A \cup B$ is compact.

6. Let A and B be two compact subsets of $\mathbb{R}$. Prove that $A \cap B$ is compact.

7. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function and $f(x) \in \mathbb{Q}$ for all $x \in \mathbb{R}$. Prove that f is a constant function.

8. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Prove that $f^{-1}(A)$ is closed for every closed set $A \subseteq \mathbb{R}$. Recall:

$$f^{-1}(A) = \{x \in \mathbb{R} : f(x) \in A\} \ .$$

9. Define $f : \mathbb{R} \to \mathbb{R}$ via

$$f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} .$$

Prove that f is continuous on all of $\mathbb{R}$.